MAT 336 Document: Descartes and "cartesian coordinates."

This extract from the 1637 edition of *La Géometrie* shows the first published instance (I believe) of Descartes' use of x and y to locate a point in the plane.

The context is the discussion of "classes" of curves. Descartes wants to show that the distinction between curves that can be drawn with ruler and compass and those that require more complicated mechanisms is artificial, and that what matters is the class (*genre*) of the curve, which depends on the degree of the equation defining it.

In this example, the mechanism is a ruler anchored at a point G on the horizontal axis, and a right triangle KLN constrained to slide along the vertical axis (KL lies along the axis, N is to the left), in such a manner that the right-angle vertex L is always on the ruler. As the ruler turns about G its intersection C with the extension of KN describes the curve in question.

Here is Descartes' text, as published in *The Geometry of René Descartes*, David Eugene Smith and Marcia L. Latham, Dover Publications, 1954 (with some minor edits).

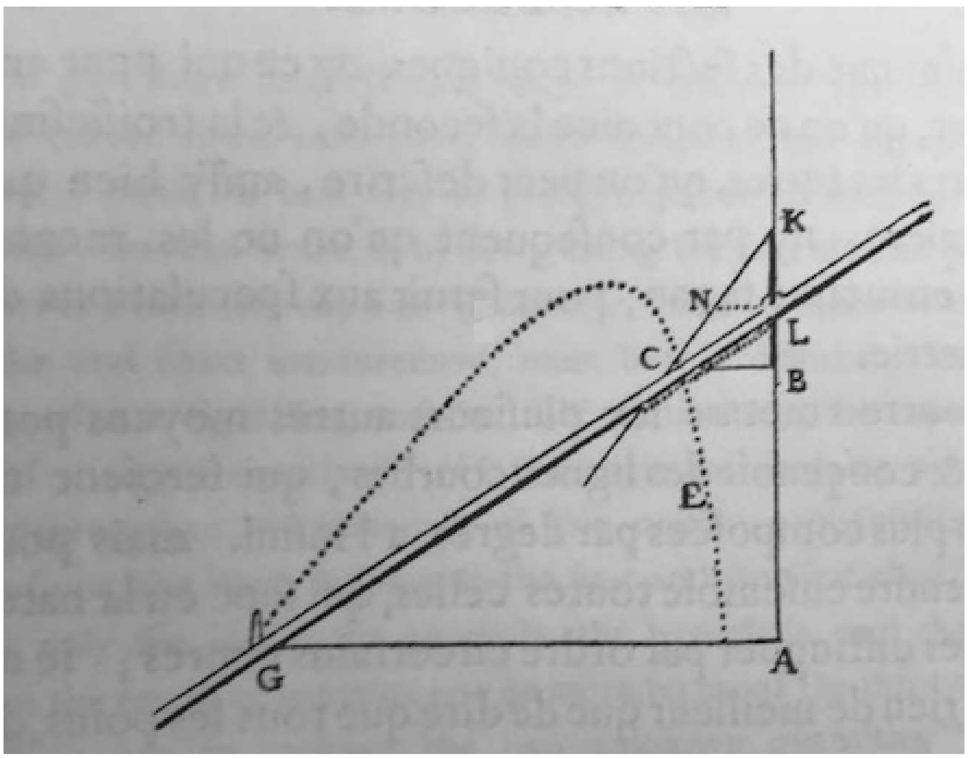
"Suppose the curve EC to be described by the intersection of the ruler GL and the rectilinear plane figure CNKL whose side KN is extended indefinitely in the direction of C, and which, being moved in the same plane in such a way that its side KL always coincides with some part of the line BA, imparts to the ruler GL a rotary motion about G (the ruler being hinged to the figure CNKL at L. If I wish to find out to what class this curve belongs, I choose a straight line, as AB, to which to refer all its points, and in AB I choose a point A at which to begin the investigation. [The class of the curve in independent of these choices].

"Then I take on the curve an arbitrary point, such as C, at which we will suppose the instrument applied to describe the curve. Then I draw through C the line CB parallel to GA. Since CB and BA are unknown and indeterminate quantities, I shall call one of them y and the other x. To the relation between these quantities I must consider also the known quantities which determine the description of the curve, as GA, which I shall call a; KL, which I shall call b, and NL parallel to GA, which I shall call c. Then I say that as NL is to LK, or as c is to b, so CB, or y, is to BK, which is therefore equal to

$\frac{b}{c}y$. Then BL is equal to $\frac{b}{c}y - b$, and AL is equal to $x + \frac{b}{c}y - b$. Moreover, as CB is to LB, that is, as y is to $\frac{b}{c}y - b$, so AG or a is to LA or $x + \frac{b}{c}y - b$. Multiplying the second by the third, we get $\frac{ab}{c}y - ab$ equal to $xy + \frac{b}{c}y^2 - by$, which is obtained by multiplying the first by the last. Therefore the required equation is

$$y^2 = cy - \frac{cx}{b}y + ay - ac.$$

From this equation we see that the curve EC belongs to the first class, it being, in fact, a hyperbola."



Note that the analysis depends on two sets of similar triangles: KLN / KBC and LBC / LAG.